

# Equations

## General Rules

- Isolate  $x$
- Follow PEMDAS
- If you do to one side, do to both
- Simplify, unless trig verify, in which you complicate.

## Factor

Final goal is zero factor property  $(a)(b) = 0 \quad a = 0 \quad b = 0$

difference of squares

$$[ \begin{aligned} & \cdot a^2 - b^2 = (a+b)(a-b) \\ & \cdot a^3 + b^3 = (a+b)(a^2 - ab + b^2) \\ & \cdot a^3 - b^3 = (a-b)(a^2 + ab + b^2) \end{aligned} ]$$

difference of cubes

$$ax^2 + bx + c$$

↳ product of  $a \cdot c$  should have factors which sum is equal to  $b$ .

↳ If above doesn't work, use quadratic formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## Inverse

- To find inverse, switch  $x$  and  $y$  in equation
- To prove two equations are inverses, plug each into the other, if just  $x$  is left, they are inverses.

## Logarithms:

$X = b^y \rightarrow$  exponential function  $b > 0$   
 $x > 0$

$\log_b X = y \rightarrow$  Logarithm

Remember!

$$\log X = \log_{10} X$$

$$\ln X = \log_e X \quad e \approx 2.72$$

$$\log_2 X - \log_2 Y = \log_2 \left( \frac{X}{Y} \right)$$

$$\log_2 X + \log_2 Y = \log_2 (XY)$$

$$\log_2 X^2 = 2 \log_2 X$$

$$\log_2 1 = 0$$

$$\log_2 2 = 1$$

$$\log_2 2^2 = 2$$

$$2^{\log_2 X} = X$$

## Simplifying Techniques (if you get stuck):

• Multiply by a form of one (be careful of extraneous solutions)

• Don't leave fractions in denominator  $\frac{\frac{x}{y}}{\frac{a}{b}} = \frac{x}{y} \cdot \frac{b}{a}$   
• form of one with

• Don't leave radicals in denominator (multiply by another radical or difference of squares)

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

• get all variables to one side  $\frac{2}{-2x}x + 5 = \frac{3x}{-2x} \rightarrow x = 5$

• factor if you can

## Extraneous Solutions

- If you get an answer that makes the original equation untrue, you have an extraneous solution.
- Result in a whole when equation is graphed.

## Synthetic & Long Division

- For synthetic division take the coefficients and put them inside of the "L" shaped chart
- Inside of the "L" shaped chart you put the value that makes  $x = 0$
- The first column drops straight down into the "L" shape
- You multiply the number on the side of the "L" shape by the number that was dropped to the bottom of the "L" shape
- The product of the number goes into its next column and is added to the number above it
- Then this process continues until equation is solved
- For long division you put dividends inside of the division sign
- The divisor goes outside of the division sign
- You then begin to do basic long division

# Equations Quiz

solve using synthetic and long division

1.)  $x^2 + 5x + 6 \div x - 1$

2.) Factor.  
 $70n^4 + 40n^3 + 28n^2 + 16n$

3.) Verify  
 $\tan x \cdot \frac{2}{4\cos^2 x - 2} = \frac{\sin x}{1 - 2\sin^2 x}$

4.)  $\log\left(\frac{3ab}{f^4}\right)$

5.) Find the inverse  
 $f(x) = \frac{4x}{x-2}$

# Equations Quiz Key

1.)  $1x^2 + 5x + 6 \div x - 1$

Synthetic

1	5	6
1	6	12

$X + 6 \text{ R } 12$

long

$$\begin{array}{r} X+6 \\ X-1 \overline{) X^2+5x+6} \\ \underline{-(X^2-x)} \phantom{+6} \\ +6x+6 \\ \underline{-(6x-6)} \\ +12 \end{array}$$

$= X + 6 \text{ R } 12$

\* don't forget to distribute the negative

\* in synthetic division you add the numbers in each column then ~~multiply~~ multiply by the number in the corner. Begin by adding the first number by with zero. Pay special attention to which numbers are positive and negative

2.)  $470n^4 + 40n^3 + 28n^2 + 16n$

$10n^3(7n+4) \quad 4n(7n+4)$

$(10n^3+4n)(7n+4)$

$= 2n(5n^2+2)(7n+4)$

← factor by grouping

← group together your factors

← factor a 2 out of the first parentheses

$$3.) \tan x \cdot \frac{\frac{2}{\sec x}}{4\cos^2 x - 2} = \frac{\sin x}{1 - 2\sin^2 x}$$

← simplify  $\frac{2}{\sec x}$

$$\tan x \cdot \frac{\frac{2}{1} \cdot \frac{1}{\sec x}}{4\cos^2 x - 2} =$$

← reciprocal property

$$\tan x \cdot \frac{2\cos x}{4\cos^2 x - 2} =$$

← factor out 2

$$\tan x \cdot \frac{(2)(\cos x)}{2(2\cos^2 x - 1)}$$

$$\tan x \cdot \frac{(2)(\cos x)}{(2)(2\cos^2 x - 1)} =$$

← eliminate because they are connected by multiplication

$$\tan x \cdot \frac{\cos x}{\cos 2x} =$$

← Double angle

reciprocal

$$\tan x \cdot \frac{\cos x}{1 - 2\sin^2 x} =$$

← simplify

$$\frac{\sin x}{\cos x} \cdot \frac{\cos x}{1 - 2\sin^2 x} =$$

↑  
finalize

common mistakes include forgetting simplifying techniques, if all are remembered there is little room for error.

$$4.) \log\left(\frac{3ab}{f^4}\right)$$

Bilal  
Hagan  
Smith

$$\log 3 + \log a + \log b - 4\log f$$

common mistakes include not writing the final form in the correct order, or putting the variables/numbers in the wrong place (ie:  $3\log$  instead of  $\log 3$ )

find inverse of

$$5.) y = \frac{4x}{x-2}$$

switch x & y

$$\frac{y-2}{1} \cdot x = \frac{4y}{x-2} \cdot \frac{y-2}{1}$$

get y out of denom

$$(y-2)x = 4y$$

distribute x

$$yx - 2x = 4y$$

get the y variables on one side

$$-2x = 4y - yx$$

factor out y

$$\frac{-2x}{4-x} = \frac{y(4-x)}{4-x}$$

isolate y

$$\boxed{y = \frac{-2}{4-x}}$$

remember that having xy is ok if you can factor it out later by getting all y variables on one side.